APPLICATION OF FUZZY SETS IN LATTICE THEORY

Synopsis

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Introduction

Ever since Zadeh introduced the concepts of a fuzzy set, the philosophy behind this notion has permeated various disciplines of human knowledge including those of logic and reasoning, which is the foundation stone of all mathematical science. Among various branches of pure and applied mathematics abstract algebra was one of the first few subjects where research was carried out using the notion of fuzzy sets. In 1971 A. Rosenfeld extended the notion of group theory to introduce a new discipline of fuzzy groups. A similar treatment in the field of lattice theory was carried out by B. Yung and W. Wu in 1990. After that N.Ajmal and K.V.Thomas systematically developed the theory of fuzzy sublattice.

Fuzzy set theory is a powerful tool to model imprecise and vague situation where exact analysis is either difficult or impossible. But in certain situations the traditional [0, 1] based fuzzy set is not always adequate. For example it does not distinguish between the
situations in which we know nothing about a certain statement and a situation in which we have exactly as many arguments in favor of this statement as we have against it.

Intuitionistic fuzzy set (in short IFS) introduced by K.T. Atanassov in 1983 as a generalization of fuzzy set theory enables us to describe this difference. Intuitionistic fuzzy sets give us the possibility to model hesitation and uncertainty by using an additional degree. In intuitionistic fuzzy sets to each element of the universe X we assign both a degree of membership and a non-membership degree with the requirement that sum of these two less than or equal to one. Basically intuitionistic fuzzy sets based models may be adequate in situations when we face human testimonies, opinions, etc. involving answers of the type yes, no, does not apply. The application of intuitionistic fuzzy sets instead of fuzzy sets means the introduction of another degree of freedom into a set description. Such a generalization of fuzzy sets gives us an additional possibility to represent imperfect knowledge what leads to describing many real problems in a more adequate way.

Intuitionistic fuzzy sets have been found to be very useful in diversely applied areas of science and technology. Extensive applications of the intuitionistic fuzzy set theory have been found in various fields such as logic programming, medical diagnosis, decision making problems and microelectronic fault analysis. Also IFS theory is conveniently and successfully applied in Abstract Algebra. In 1996 R. Biswas introduced intuitionistic fuzzy subgroups and its properties. Further K. H. Kim et.al applied the notion of intuitionistic fuzzy sets to semigroups and studied the notions of interior ideals, bi-ideals, Q-fuzzy semi prime ideals etc. B. Banerjee and D.K. Basent introduced intuitionistic
fuzzy subrings and ideals. W. Dudek introduced and study intuitionistic fuzzy h-ideals of hemirings.

In the light of these developments, we start with fuzzy sublattices and ideals of a lattice. Further we generalize these concepts using the notion of IFS. Also we introduced various operations between intuitionistic fuzzy ideals of a lattice and study their properties under these operations. The notions of ideal of an intuitionistic fuzzy sublattice and residual of ideals are introduced. We thoroughly discuss the concepts of intuitionistic fuzzy equivalence and congruence relations by generalizing their usual definition.

In 1975 Zadeh made an extension of the concept of fuzzy set by an interval-valued fuzzy set. P. P. Ming and L. Y. Ming introduced the concept of quasi-coincidence of a fuzzy point with a fuzzy subset. Recently D. S. Lee and C. H. Park introduced the notion of interval valued $(\in, \in \vee q)$-fuzzy subring and ideal, and investigated their properties. Using the ‘belongs to’ relation and ‘quasi-coincidence’ between intuitionistic fuzzy points and intuitionistic fuzzy sets, Y. B. Jun introduced the concept of $(\phi, \psi)$-intuitionistic fuzzy subgroup. Later H. Hedayati introduced interval valued intuitionistic fuzzy substructures in semirings with respect to $t$- norm $T$ and $s$- norm $S$ and their characterization based on level subsets. In view of these developments we studied sublattices and ideals of a lattice under the notions of ‘belongs to’ and ‘quasi-coincidence’ in the IFS setting.

Rough set theory has been proposed by Pawlak as a tool for dealing with the vagueness and granularity in information systems. It is a method to conceptualize, organize and analyze various types of data in data mining. Rough set theory has found practical
application in many areas such as knowledge discovery, machine learning, data analysis, etc. Also there is connection between rough sets and algebraic systems. Some authors for example Biswas and Nanda introduced the notion of rough subgroups. Kuroki introduced rough ideals in a semigroup, Kuroki and Wang studied properties of lower and upper approximation with respect to normal subgroups. B.Davaaz introduced rough subring and ideals with respect to an ideal of a ring. In this work, we studied the rough set approximation of a set with respect to an ideal of a lattice and introduced rough sublattice (ideals and prime ideals). Theories of rough sets and fuzzy sets are related but distinct and complementary theories. Integration of these two theories play significant role in the development of a new theory of fuzzy rough sets. Dubois and Prade proposed the notions of rough fuzzy sets and fuzzy rough sets. In the light of the developments made sofar we introduced the concepts of rough fuzzy sublattices, rough fuzzy ideal and prime ideal and studied rough sublattice and ideal in the frame work of IFS.

Chapter wise summary

The whole thesis is divided in to seven chapters and each chapter is further subdivided into a number of sections.

In chapter 1, a brief history of the theory and development of fuzzy sets, intuitionistic fuzzy sets and rough sets are provided. This chapter also presents a summary of the research work carried out in the thesis. Chapter 2 emphasizes on basic definitions, results and properties of lattices, fuzzy sets and intuitionistic fuzzy sets, which serve as a prerequisite for the research work done in the thesis.

In chapter 3, the concepts of intuitionistic fuzzy sublattice (IFL) and intuitionistic fuzzy ideal (IFI) are introduced and discussed. Firstly we gave some properties of these
concepts. As in the case of fuzzy setting, in intuitionistic fuzzy setting also the notion of level subset is found to be an important tool for establishing various lattice theoretic properties. Here we have given characterizations of intuitionistic fuzzy sublattices and intuitionistic fuzzy ideals in terms of its level subsets. In the sequel, intuitionistic fuzzy convex sublattice is defined and its the level subset characterization is provided. Also properties of intuitionistic fuzzy ideals under lattice homomorphism are studied. We defined f-invariant class of IFIs and established a correspondence between the IFIs of a lattice which are f-invariant and IFIs of its homomorphic image. This chapter is concluded with the result that the homomorphism image of intuitionistic fuzzy prime ideal with supremum and infimum property or f-invariant property is again an intuitionistic fuzzy prime ideal.

In chapter 4, some operations on intuitionistic fuzzy sets are introduced and based on that properties of intuitionistic fuzzy ideals under these operations are studied. Different characterizations of intuitionistic fuzzy sublattices and intuitionistic fuzzy ideals are given in terms of these operations. Using these operations for certain subclass of the class of intuitionistic fuzzy ideals, the join of two intuitionistic fuzzy ideals is constructed. This leads to the formation of various lattices and sublattices of intuitionistic fuzzy ideals. In the next section we defined the intuitionistic fuzzy ideal of an intuitionistic fuzzy lattice and studied their properties under the operations defined in the first section. Moreover the concept of quotient (or residual) of ideals of an intuitionistic fuzzy lattice is introduced and proved that it is again an ideal of the intuitionistic fuzzy lattice.

Chapter 5 is devoted to a detailed study of intuitionistic fuzzy equivalence and congruence relations including their lattice structures. Firstly we slightly modify their
definitions and referred to it as (t, k) equivalence and congruence relations. We studied certain properties of them and given the characterization of (t, k) equivalence and congruence in terms of their level subsets and proved that the class of all intuitionistic fuzzy congruence relations forms a distributive lattice. We established the relationship between intuitionistic fuzzy congruence and intuitionistic fuzzy ideals in a distributive lattice and using the notion of upper and lower level subsets of these concepts we proved that in a generalized Boolean algebra their lattices are isomorphic.

In this chapter we also defined an intuitionistic fuzzy partition called (t, k)-partition and a canonical one-to-one correspondence between (t, k)-partitions and (t, k)-equivalences is established. Moreover it is verified that in a lattice, (t, k)-partitions with respect to a (t, k)-congruence constitute a lattice. This lattice is called the quotient lattice with respect to a given (t, k) - congruence relation. We also studied the concept of quotient of an IFL relative to ordinary congruence relation and proved the intuitionistic fuzzy version of fundamental homomorphism theorem.

A systematic development of the theory of interval valued intuitionistic fuzzy sublattices is carried out in chapter 6. Firstly we defined interval valued intuitionistic fuzzy sublattices and ideals and some properties of their level subsets. Also we gave some properties of interval valued intuitionistic fuzzy ideals under lattice homomorphism. Further based on the concepts of ‘belongingness’ and ‘quasi-coincidence’ of an interval valued intuitionistic fuzzy point with an interval valued intuitionistic fuzzy set, a new type of intuitionistic fuzzy sublattice (ideal) is provided and called \((\in_E, \in \vee q)\) intuitionistic fuzzy sublattice (ideal). Properties of these new classes of
sublattices and ideals are studied and their characterizations in terms of level subsets are established.

Finally, in chapter 7 we discussed about rough sets in lattice theory. In a distributive lattice there exist a one to one correspondence between ideals and congruence relations. Using this notion we defined the rough set approximation of a set with respect to an ideal of a lattice and introduced rough sublattice (ideals and prime ideals). Also some properties of the lower and upper approximations of a set in a lattice are studied. Moreover we introduced the concepts of rough fuzzy sublattices, (ideal and prime ideal) in a lattice and their properties are studied. Further we discussed the homomorphisms images of upper and lower rough ideals (prime ideals). The developments achieved so far allows as to give a rough approximation of a intuitionistic fuzzy set and which introduces rough intuitionistic fuzzy sublattices, ideals etc. A detailed study of these concepts including intuitionistic fuzzy rough sets, intuitionistic fuzzy rough sublattices, (ideals) are provided.

The main objectives of this thesis are,

(1) To enrich the existing literature on fuzzy lattice theory.

(2) To extend some basis concepts and results of lattice theory to the intuitionistic fuzzy setting in a unified way.

(3) To improve and unify the studies of fuzzy sets in lattice theory.

The subject matter of the thesis depends on the following papers:


(4) K.V Thomas and Latha.S.Nair, Quotient of ideals of an intuitionistic fuzzy lattice, 
*Advances in fuzzy systems* vol.2010 article ID 781672, 8pages.

(5) K.V Thomas and Latha.S.Nair, Rough Intuitionistic fuzzy sets in a lattice. 


(9) K.V Thomas and Latha.S.Nair, Interval valued Intuitionistic fuzzy ideals in a Lattice. *International journal of fuzzy systems*. (Communicated)

REFERENCES


